

# Quantization of the tachyonic field

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A consistent quantization scheme for imaginary-mass field is proposed. It is related to an appropriate choice of the synchronization procedure (definition of time), which guarantee an absolute causality. In that formulation a possible existence of field excitations (tachyons) distinguish an inertial frame (tachyon privileged frame of reference) *via* spontaneous breaking of the so called synchronization group.

## I. INTRODUCTION

Some attention in the literature over last decades, related to the question of existence of faster-than-light particles, has been lacking in view of the apparent conflict with the causality principle. Irrespective of an attempt to reconcile the notion of superluminal objects with causality on the classical and/or semiclassical level [1–3], it is commonly believed that there is no respectable tachyonic quantum field theory at present (for some efforts in this direction see [4,5]).

However, in the last time we observe a return of interest in tachyons. This is related to some recent experimental data [6,7] indicating that the square of the muon neutrino mass seems to be negative<sup>1</sup>. Experimental data for the electron neutrino prefer also a negative value for its mass square, but there are not so transparent [6].

On the other hand the admittance of space-like four-momentum eigenstates can possibly improve situation in quantum field theory by the weakness of the spectral condition. Furthermore non-localizability of tachyonic modes [10] may moderate QFT divergences. It is also noticeable that a tachyonic condensate is an immanent point of superstring models [11–16].

This paper is motivated by the problems mentioned above. Here we make the first step in this direction by a consistent quantization of a scalar imaginary-mass field. Our quantization scheme is related to a nonstandard procedure of synchronization of clocks proposed in [17–20]. This procedure allows us to introduce the notion of a coordinate time appropriate to the definition of the universal notion of causality in agreement with special relativity. The main results can be summarized as follows:

- The relativity principle is formulated in the framework of a nonstandard synchronization scheme (the Chang–Thangherlini (CT) scheme). The absolute causality holds for all kinds of events (time-like, light-like, space-like).
- For bradyons and luxons our scheme is fully equivalent to the standard formulation of special relativity.
- For tachyons it is possible to formulate covariantly proper initial conditions.
- There exists a (covariant) lower bound of energy for tachyons.
- The paradox of “transcendental” tachyons is solved.
- Tachyonic field can be consistently quantized using the CT synchronization scheme.
- The familiar “reinterpretation principle” [21] cannot be unitarily implemented on the quantum level; moreover there is no such necessity.
- Tachyons distinguish a preferred frame *via* mechanism of the spontaneous symmetry breaking [22].

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<sup>1</sup>In the paper by Chodos *et al.* [8] it has been suggested that the muon neutrino might possible be a fermionic tachyon. (See also [9]).

## II. PRELIMINARIES

As is well known, in the standard framework of the special relativity, space-like geodesics do not have their physical counterparts. This is an immediate consequence of the assumed causality principle which admits time-like and light-like trajectories only.

In the papers by Terletsky [23], Tanaka [24], Sudarshan *et al.* [25], Recami *et al.* [1–3] and Feinberg [21] the causality problem has been reexamined and a physical interpretation of space-like trajectories was introduced. However, every proposed solution raises new unanswered questions of the physical or mathematical nature [26]. The difficulties are specially frustrating on the quantum level [4,5,27]. It is rather evident that a consistent description of tachyons lies in a proper extension of the causality principle. Note that interpretation of the space-like world lines as physically admissible tachyonic trajectories favour the constant-time initial hyperplanes. This follows from the fact that only such surfaces intersect each world line with locally nonvanishing slope once and only once. Unfortunately, the instant-time hyperplane is *not a Lorentz-covariant notion* which is just the source of many troubles with causality.

The first step toward a solution of this problem can be found in the papers by Chang [18,19,28], who introduced four-dimensional version of the Tangherlini transformations [17], termed the Generalized Galilean Transformations (GGT). In [20] it was shown that GGT, extended to form a group, are hidden (nonlinear) form of the Lorentz group transformations with  $SO(3)$  as a stability subgroup. Moreover, a difference with the standard formalism lies in a nonstandard choice of the synchronization procedure. As a consequence a constant-time hyperplane is a covariant notion. In the following we will call this procedure of synchronization the *Chang–Tangherlini synchronization scheme*.

It is important to stress the following two well known facts from special relativity: (a) the definition of a coordinate time depends on the synchronization scheme [29–31], (b) synchronization scheme is a convention, because no experimental procedure exists which makes it possible to determine the one-way velocity of light without use of superluminal signals [32]. Notice that a choice of a synchronization scheme *does not affect the assumptions of special relativity but evidently it can change the causality notion*, depending on the definition of the coordinate time.

Following Einstein, intrasystemic synchronization of clocks in their “setting” (zero) requires a definitional or conventional stipulation (for discussion see Jammer [32] and Sjödin [33]). Really, to determine one-way light speed it is necessary to use synchronized clocks (at rest) in their “setting” (zero)<sup>2</sup>. On the other hand to synchronize clocks we should know the one-way light velocity. Thus we have a logical loophole. In other words no experimental procedure exists (if we exclude superluminal signals) which makes possible to determine unambiguously and without any convention the one-way velocity of light. Consequently, an *operational meaning has the average value of the light velocity around closed paths only*. This statement is known as the conventionality thesis [32]. Following Reichenbach [29], two clocks A and B stationary in the points A and B of an inertial frame are defined as being synchronous with help of light signals if  $t_B = t_A + \varepsilon_{AB}(t'_A - t_A)$ . Here  $t_A$  is the emission time of light signal at point A as measured by clock A,  $t_B$  is the reception-reflection time at point B as measured by clock B and  $t'_A$  is the reception time of this light signal at point A as measured by clock A. The so called synchronization coefficient  $\varepsilon_{AB}$  is an arbitrary number from the open interval (0, 1). In principle it can vary from point to point. The only conditions for  $\varepsilon_{AB}$  follow from the requirements of symmetry and transitivity of the synchronization relation. Note that  $\varepsilon_{AB} = 1 - \varepsilon_{BA}$ . The one-way velocities of light from A to B ( $c_{AB}$ ) and from B to A ( $c_{BA}$ ) are given by

$$c_{AB} = \frac{c}{2\varepsilon_{AB}}, \quad c_{BA} = \frac{c}{2\varepsilon_{BA}}.$$

Here  $c$  is the round-trip average value of the light velocity. In standard synchronization  $\varepsilon_{AB} = \frac{1}{2}$  and consequently  $c = c_{AB}$  for each pair A, B.

The conventionality thesis states that from the operational point of view the choice of a fixed set of the coefficients  $\varepsilon$  is a convention. However, the explicit form of the Lorentz transformations will be  $\varepsilon$ -dependent in general. The question arises: Are equivalent notions of causality connected with different synchronization schemes? As we shall see throughout this work the answer is *negative* if we admit tachyonic world lines. In other words, the causality requirement, logically independent of the requirement of the Lorentz covariance, can contradict the conventionality thesis and consequently it can prefer a definite synchronization scheme, namely CT scheme.

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<sup>2</sup>Obviously it is possible to synchronize clocks in their rate without knowledge of the one-way light speed [34].

### III. THE CHANG–TANGHERLINI SYNCHRONIZATION

As was mentioned in Section II, in the paper by Tangherlini [17] a family of inertial frames in  $1+1$  dimensional space of events was introduced with the help of transformations which connect the time coordinates by a simple (velocity dependent) rescaling. This construction was generalized to the  $1+3$  dimensions by Chang [18,19]. As was shown in the paper [20], the Chang–Tangherlini inertial frames can be related by a group of transformations isomorphic to the orthochronous Lorentz group. Moreover, the coordinate transformations should be supplemented by transformations of a vector-parameter interpreted as the velocity of a privileged frame. It was also shown that the above family of frames is equivalent to the Einstein–Lorentz one; (in a contrast to the interpretation in [18,19]). A difference lies in another synchronization procedure for clocks [20].

In this Section we derive a realization of the Lorentz group given in [20] in a systematic way [22]. Furthermore we will discuss physical content of our formalism.

Let us start with a simple observation that the description of a family of (relativistic) inertial frames in the Minkowski space-time is not so convenient and natural. Instead, it seems that the geometrical notion of bundle of frames is more natural. Base space is identified with the space of velocities; each velocity marks out a coordinate frame. Indeed, from the point of view of an observer (in a fixed inertial frame) all inertial frames are labelled by their velocities with respect to him. Therefore, in principle, to define the transformation rules between frames, we can use, except of coordinates, also this vector-parameter, possibly related to velocities of frames with respect to a (formally) distinguished observer. Because we adopt Lorentz covariance, we can use a time-like fourvelocity  $u_E$ ; subscript  $E$  means Einstein–Poincaré synchronization<sup>3</sup> (EP synchronization) i.e. we adopt, at this moment, the standard transformation law for  $u_E$

$$u'_E = \Lambda u_E$$

where  $\Lambda$  is an element of the Lorentz group  $L$ .

Below we list our main assumptions:

1. Coordinate frames are related by a set of transformations isomorphic to the Lorentz group (**Lorentz covariance**).
2. The average value of the light speed over closed paths is constant ( $c$ ) for all inertial observers (**constancy of the round-trip light velocity**).
3. With respect to the rotations  $x^0$  and  $\vec{x}$  transform as  $SO(3)$  singlet and triplet respectively (**isotropy**).
4. Transformations are linear with respect to the coordinates (**affinity**).
5. (\*) We admit an additional set of parameters  $u_E$  (**the base space** for a bundle of inertial frames).
6. (\*) Instant-time hyperplanes are covariant under coordinate transformations (**absolute causality**).

We see that assumptions labelled by star  $*$  are essentially new.

#### A. Derivation of the Lorentz group transformation rules in the CT synchronization

According to our assumptions, transformations between two coordinate frames  $x^\mu$  and  $x'^\mu$  have the following form

$$x'(u'_E) = D(\Lambda, u_E)x(u_E). \quad (1)$$

Here  $D(\Lambda, u_E)$  is a real (invertible)  $4 \times 4$  matrix,  $\Lambda$  belongs to the Lorentz group and  $u_E^\mu$  is assumed to be a Lorentz four-vector, i.e.

$$u'_E = \Lambda u_E, \quad u_E^2 = c^2 > 0. \quad (2)$$

The physical meaning of  $u_E^\mu$  will be explained later. It is easy to verify that the transformations (1–2) constitute a realization of the Lorentz group if the following composition law holds

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<sup>3</sup>In the papers by Chang [18,19,28] it was used some kinematical objects with an unproper physical interpretation [35,36]. For this reason we should be precise in the nomenclature related to different synchronizations.

$$D(\Lambda_2, \Lambda_1 u_E) D(\Lambda_1, u_E) = D(\Lambda_2 \Lambda_1, u_E). \quad (3)$$

Now we demand that  $(x^\mu) \equiv (x^0, \vec{x})$  transform under subgroup of rotations as singlet + triplet (isotropy condition) i.e. for  $R \in SO(3)$

$$\Omega \equiv D(R, u_E) = \begin{pmatrix} 1 & 0 \\ 0 & R \end{pmatrix}. \quad (4)$$

From eqs. (1–3) we see that the identity and the inverse element have the form

$$I = D(I, u_E), \quad (5a)$$

$$D^{-1}(\Lambda, u_E) = D(\Lambda^{-1}, \Lambda u_E). \quad (5b)$$

Using the familiar Wigner trick we obtain that

$$D(\Lambda, u_E) = T(\Lambda u_E) \Lambda T^{-1}(u_E), \quad (6)$$

where the real matrix  $T(u_E)$  is given by

$$T(u_E) = D(L_{u_E}, \tilde{u}_E) L_{u_E}^{-1}. \quad (7)$$

Here  $\tilde{u}_E = (c, 0, 0, 0)$  and  $L_{u_E}$  is the boost matrix:  $u_E = L_{u_E} \tilde{u}_E$ . We use the following parametrization of the matrix  $L_{u_E}$

$$L_{u_E} = \left( \begin{array}{c|c} \frac{u_E^0}{c} & \frac{\vec{u}_E^T}{c} \\ \hline \frac{\vec{u}_E}{c} & I + \frac{\vec{u}_E \otimes \vec{u}_E^T}{c^2 \left(1 + \frac{u_E^0}{c}\right)} \end{array} \right).$$

Note that the transformations (1–2) leave the bilinear form  $x^T(u_E) g(u_E) x(u_E)$ , where the symmetric tensor  $g(u_E)$  reads

$$g(u_E) = (T(u_E) \eta T^T(u_E))^{-1}, \quad (8)$$

invariant. Here  $\eta$  is the Minkowski tensor and the superscript  $^T$  means transposition.

Now we determine the matrix  $T(u_E)$ . To do this we note that under rotations

$$T(\Omega u_E) = \Omega T(u_E) \Omega^{-1},$$

so the most general form of  $T(u_E)$  reads

$$T(u_E) = \left( \begin{array}{c|c} a(u_E^0) & b(u_E^0) \vec{u}_E^T \\ \hline d(u_E^0) \vec{u}_E & e(u_E^0) I + (\vec{u}_E \otimes \vec{u}_E^T) f(u_E^0) \end{array} \right), \quad (9)$$

where  $a, b, d, e$  and  $f$  are some functions of  $u_E^0$ . Inserting eq. (9) into eq. (8) we can express the metric tensor  $g(u_E)$  by  $a, b, d, e$  and  $f$ . In a three dimensional flat subspace we can use an orthogonal frame (i.e.  $(g^{-1})^{ik} = -\delta^{ik}$ ;  $i, k = 1, 2, 3$ ), so we obtain

$$e(u_E^0) = 1, \quad d^2 = (2 - f \vec{u}_E^2) f. \quad (10)$$

Furthermore, from the equation of null geodesics,  $dx^T g dx = 0$ , we deduce that the light velocity  $\vec{c}$  in the direction  $\vec{n}$  ( $\vec{n}^2 = 1$ ) is of the form

$$\vec{c} = c \vec{n} \left( \sqrt{\alpha + \beta^2 \vec{u}_E^2} - \beta \vec{u}_E \vec{n} \right)^{-1}, \quad (11)$$

where  $\alpha = a^2 - b^2 \vec{u}_E^2$ ,  $\beta = ad - b(1 + f \vec{u}_E^2)$ . From eq. (11) we see that the synchronization convention depends on the functions  $\alpha$  and  $\beta$  only. Now, because  $a$ ,  $b$  and  $d$  can be expressed as functions of  $\alpha$ ,  $\beta$  and  $f$  and we are interested in *essentially different synchronizations* only, we can choose

$$f = 0, \quad (12)$$

so

$$d = 0, \quad \beta = -b, \quad \alpha = a^2 - b^2 \vec{u}_E^2. \quad (13)$$

Finally, from (11–13) the average value of  $|\vec{c}|$  over a closed path is equal to

$$\langle |\vec{c}| \rangle_{\text{cl. path}} = \frac{c}{a}.$$

Because *we demand that the round-trip light velocity* ( $\langle |\vec{c}| \rangle_{\text{cl. path}} = c$ ) *be constant*, we obtain

$$a = 1. \quad (14)$$

Summarizing,  $T(u_E)$  has the form

$$T(u_E) = \left( \begin{array}{c|c} 1 & b(u_E^0) \vec{u}_E^T \\ \hline 0 & I \end{array} \right), \quad (15)$$

while the light velocity

$$\vec{c} = c \vec{n} (1 + b \vec{u}_E \vec{n})^{-1}, \quad (16)$$

so the Reichenbach coefficient reads

$$\varepsilon(\vec{n}, \vec{u}_E) = \frac{1}{2} (1 + b \vec{u}_E \vec{n}). \quad (17)$$

In special relativity the function  $b(u_E^0)$  distinguishes between different synchronizations. Choosing  $b(u_E^0) = 0$  we obtain  $\vec{c} = c \vec{n}$ ,  $\varepsilon = \frac{1}{2}$  and the standard transformation rules for coordinates:  $x' = \Lambda x$ . On the other hand, if we demand that the instant-time hyperplane  $x^0 = \text{constant}$  be an invariant notion, i.e. that  $x'^0 = D(\Lambda, u_E)^0_0 x^0$  so  $D(\Lambda, u_E)^0_k = 0$ , then from eqs. (6, 15) we have

$$b(u_E^0) = -\frac{1}{u_E^0}. \quad (18)$$

In the following we restrict ourselves to the above case defined by eq. (18). Notice that  $\vec{u}_E/u_E^0$  can be expressed by a three-velocity  $\vec{\sigma}_E$

$$\frac{\vec{u}_E}{u_E^0} = \frac{\vec{\sigma}_E}{c} \quad (19)$$

with  $0 \leq |\vec{\sigma}_E| < c$ . Therefore

$$T(u_E) \equiv T(\vec{\sigma}_E) = \left( \begin{array}{c|c} 1 & -\frac{\vec{\sigma}_E^T}{c} \\ \hline 0 & I \end{array} \right). \quad (20)$$

Thus we have determined the form of the transformation law (1) in this case. Now, according to our interpretation of the freedom in the Lorentz group realization as the synchronization convention freedom, there should exist a relationship between  $x^\mu$  coordinates and the Einstein–Poincaré ones. In fact, the matrix  $T$  relates both synchronizations *via* the formula

$$x = T(\vec{\sigma}_E) x_E. \quad (21)$$

Explicitly:

$$x^0 = x_E^0 - \frac{\vec{\sigma}_E}{c} \vec{x}_E, \quad (22a)$$

$$\vec{x} = \vec{x}_E. \quad (22b)$$

It is easy to check that  $x_E$  transforms according to the standard law i.e.

$$x'_E = \Lambda x_E. \quad (23)$$

Now, by means of eq. (22) we obtain analogous relations between differentials

$$dx^0 = dx_E^0 - \frac{\vec{\sigma}_E}{c} d\vec{x}_E, \quad (24a)$$

$$d\vec{x} = d\vec{x}_E, \quad (24b)$$

and consequently interrelations between velocities in both synchronizations; namely

$$\vec{v} = \frac{\vec{v}_E}{1 - \frac{\vec{v}_E \vec{\sigma}_E}{c^2}}, \quad (25)$$

$$\vec{v}_E = \frac{\vec{v}}{1 + \frac{\vec{\sigma} \vec{v}}{c^2} \gamma_0^{-2}}. \quad (26)$$

Here  $\vec{\sigma}$  is the  $\vec{\sigma}_E$  velocity in the CT synchronization, i.e.

$$\vec{\sigma} = \frac{\vec{\sigma}_E}{1 - \left(\frac{\vec{\sigma}_E}{c}\right)^2}, \quad (27)$$

while  $\gamma_0$  is defined as

$$\gamma_0 = \left[ \frac{1}{2} \left( 1 + \sqrt{1 + \left(\frac{2\vec{\sigma}}{c}\right)^2} \right) \right]^{1/2}. \quad (28)$$

In the following we use also the quantity  $\gamma(\vec{v})$  defined as follows

$$\gamma(\vec{v}) = \left| \left( 1 + \frac{\vec{\sigma} \vec{v}}{c^2} \gamma_0^{-2} \right)^2 - \left( \frac{\vec{v}}{c} \right)^2 \right|^{1/2}. \quad (29)$$

Now, taking into account eqs. (16, 18, 19) we see that the light velocity  $\vec{c}$  in the direction of a unit vector  $\vec{n}$  reads

$$\vec{c} = \frac{c\vec{n}}{1 - \frac{\vec{n} \vec{\sigma}_E}{c}}, \quad \text{labelD12} \quad (30)$$

i.e. in terms of  $\vec{\sigma}$  (see eq. (27))

$$\vec{c} = \frac{c\vec{n}}{1 - \frac{\vec{n} \vec{\sigma}}{c} \gamma_0^{-2}}, \quad (31)$$

so

$$\varepsilon(\vec{n}, \vec{\sigma}) = \frac{1}{2} \left( 1 - \frac{\vec{n} \vec{\sigma}}{c} \gamma_0^{-2} \right). \quad (32)$$

We call the synchronization scheme defined by the above choice of the Reichenbach coefficients the *Chang–Tangherlini synchronization*. In terms of  $\vec{\sigma}$ , the matrix  $T$  reads

$$T(\vec{\sigma}) = \left( \begin{array}{c|c} 1 & -\frac{\vec{\sigma}^T}{c}\gamma_0^{-2} \\ \hline 0 & I \end{array} \right). \quad (33)$$

Let us return to the transformation laws (1) and (2). By means of the formulas (6, 26) and (33) we can deduce the following explicit form of the Lorentz group transformations (1–2) in the CT synchronization [20,22]

### Boosts

$$x'^0 = \gamma x^0, \quad (34a)$$

$$\vec{x}' = \vec{x} + \frac{\vec{V}}{c} \left[ \frac{\vec{V} \cdot \vec{x}}{c \left( \gamma + \sqrt{\gamma^2 + \left( \frac{\vec{V}}{c} \right)^2} \right)} - \frac{\vec{\sigma} \cdot \vec{x}}{c\gamma_0} - x^0 \right] \gamma^{-1}, \quad (34b)$$

$$\vec{\sigma}' = \vec{\sigma} \gamma^{-1} + \vec{V} \gamma^{-2} \left[ \frac{\vec{V} \cdot \vec{\sigma}}{c^2 \left( \gamma + \sqrt{\gamma^2 + \left( \frac{\vec{V}}{c} \right)^2} \right)} - \left( \frac{\vec{\sigma}}{c} \right)^2 \gamma_0^{-1} - 1 \right]. \quad (34c)$$

Here  $\gamma = \gamma(\vec{V})$  has the form (29).

### Rotations (compare with (4))

$$x'^0 = x^0, \quad (35a)$$

$$\vec{x}' = R\vec{x}, \quad (35b)$$

$$\vec{\sigma}' = R\vec{\sigma}. \quad (35c)$$

It is easy to see that a vector  $\vec{V}$  appearing in the transformations rules (34) is the relative velocity of the frame  $x'$  with respect to  $x$ , measured in the CT synchronization. Moreover, from (34–35) we can deduce the meaning of the vector-parameter  $\vec{\sigma}$ ; namely  $\vec{\sigma}$  is a velocity of a fixed (formally privileged) frame as measured by an observer which uses the coordinates  $x(\vec{\sigma})$ .

Notice that the matrix  $D$  (eq. (1)) for Lorentz boosts reads

$$D(\vec{V}, \vec{\sigma}) = \left( \begin{array}{c|c} \gamma & 0 \\ \hline -\frac{\vec{V}}{c}\gamma^{-1} & I + \frac{\vec{V} \otimes \vec{V}^T}{c^2 \gamma \left[ \gamma + \sqrt{\gamma^2 + \frac{\vec{V}^2}{c}} \right]} - \frac{\vec{V} \otimes \vec{\sigma}^T}{c^2 \gamma \gamma_0} \end{array} \right). \quad (36)$$

For completeness, we give also the explicit form of the metric tensors  $g_{\mu\nu}(\vec{\sigma})$  and  $g^{\mu\nu}(\vec{\sigma})$ :

$$[g_{\mu\nu}(\vec{\sigma})] = \left( \begin{array}{c|c} 1 & \frac{\vec{\sigma}^T}{c} \gamma_0^{-2} \\ \hline \frac{\vec{\sigma}}{c} \gamma_0^{-2} & -I + \frac{\vec{\sigma} \otimes \vec{\sigma}^T}{c^2} \gamma_0^{-4} \end{array} \right), \quad (37a)$$

$$[g^{\mu\nu}(\vec{\sigma})] = \left( \begin{array}{c|c} \gamma_0^{-2} & \frac{\vec{\sigma}^T}{c} \gamma_0^{-2} \\ \hline \frac{\vec{\sigma}}{c} \gamma_0^{-2} & -I \end{array} \right). \quad (37b)$$

From (37b) it is evident that the three-space is the Euclidean one. Furthermore, the subset of transformations (34) defined by the condition  $\vec{\sigma} = 0$  coincides exactly with the family of the Chang–Tangherlini inertial frames [18,19].

### B. Causality and kinematics in the CT synchronization

In this subsection we discuss shortly differences and similarities of kinematical descriptions in both CT and EP synchronizations. Recall that in CT scheme *causality has an absolute meaning*. This follows from the transformation law (34) for the coordinate time:  $x^0$  is rescaled by a positive, velocity dependent factor  $\gamma$ . Thus this formalism extends the EP causality by allowing faster than light propagation. It can be made transparent if we consider the relation derived from eq. (24)

$$\frac{dx^0}{dx_E^0} = 1 - \frac{\vec{\sigma}_E \vec{v}_E}{c^2}. \quad (38)$$

For  $|\vec{v}_E| \leq c$  we have  $\frac{dx^0}{dx_E^0} > 0$ , whereas for  $|\vec{v}_E| > c$ ,  $\frac{dx^0}{dx_E^0}$  can be evidently negative, which is a consequence of an inadequacy of the EP synchronization in this situation. Notice that subluminal (superluminal) signals in the EP synchronization remain subluminal (superluminal) in the CT one too; indeed, as we see from eqs. (25, 26, ??, 31) the rate of the corresponding velocities in the same direction  $\vec{n}$  reads

$$\frac{|\vec{v}|}{|\vec{c}|} = \left| \frac{\frac{v_E}{c} - \frac{\vec{v}_E \vec{\sigma}_E}{c^2}}{1 - \frac{\vec{v}_E \vec{\sigma}_E}{c^2}} \right| < 1 \quad \text{iff} \quad \left| \frac{v_E}{c} \right| < 1.$$

Let us consider in detail a space-like four-momentum  $k^\mu$  transforming under  $D$  determined by eqs. (34–35) (see also (36)). Only in this case we can expect a real deviation from the standard description. Now, our  $k$  satisfy

$$k^2 = g_{\mu\nu}(\vec{\sigma}) k^\mu k^\nu = g^{\mu\nu}(\vec{\sigma}) k_\mu k_\nu = -\kappa^2 < 0. \quad (39)$$

Because velocity of a particle has direct physical meaning we solve the tachyonic dispersion equation (39) by means of the evident relations

$$k^\mu = \frac{\kappa}{c} w^\mu$$

with  $w^2 = -c^2$ , and

$$\frac{\vec{v}}{c} = \frac{d\vec{x}}{dx^0} = \frac{\vec{w}}{w^0}.$$

Consequently the solution of eq. (39) reads

$$k_\pm^0 = \pm \kappa \gamma^{-1}, \quad (40a)$$

$$\vec{k}_\pm = \pm \kappa \gamma^{-1} \frac{\vec{v}}{c}, \quad (40b)$$

where  $\gamma = \gamma(\vec{v})$  is given by eq. (29).



Now, by means of (37a) the covariant fourmomentum  $k_\mu$  has the form

$$k_{0\pm} = \pm\kappa\gamma^{-1} \left( 1 + \gamma_0^{-2} \frac{\vec{\sigma}\vec{v}}{c^2} \right), \quad (41a)$$

$$\vec{k}_\pm = \pm\kappa\gamma^{-1} \left[ -\frac{\vec{v}}{c} + \frac{\vec{\sigma}}{c} \gamma_0^{-2} \left( 1 + \gamma_0^{-2} \frac{\vec{\sigma}\vec{v}}{c^2} \right) \right]. \quad (41b)$$

Recall that the generators of space-time translations are covariant, so energy must be identified with  $k_0$ . To make a proper identification of energy ( $k_{0+}$  or  $k_{0-}$ ), let us analyse the above formulas with the help of convenient parameters  $\xi$ ,  $s$  and  $\varepsilon$

$$\xi = \frac{|\vec{v}|}{|\vec{c}|} \in (1, \infty), \quad (\text{for tachyons}), \quad (42a)$$

$$s = \frac{|\vec{c}|}{c} \in \left(\frac{1}{2}, \infty\right), \quad (42b)$$

$$\varepsilon = \gamma_0^{-2} \frac{\sigma}{c} \in (0, 1). \quad (42c)$$

Here  $\vec{c}$  propagates in the direction of  $\vec{v}$ , i.e.  $\vec{n}$  in eq. (31) is assumed to have the form  $\vec{n} = \vec{v}/|\vec{v}|$ . In terms of  $\xi$  and  $s$

$$k_{0\pm} = \pm\kappa \frac{1 + (s-1)\xi}{\sqrt{(\xi-1)[(2s-1)\xi+1]}}. \quad (43)$$

We see that a proper choice for tachyon energy is  $k_{0+}$ ; indeed  $k_{0+}$  has a lower bound. Moreover, this property is covariant because  $k_+^0$  is positive in that case and  $\varepsilon(k_+^0) = 1 = \text{invariant}$ . Notice also that the lowest value of energy  $k_{0\min} = \kappa(s-1)/(2s-1)^{1/2}$  depends only on the light propagation characteristics in a given frame. Thus in the CT synchronization, contrary to the EP one, tachyonic energy is bounded from below. This fact is especially important because implies stability on the quantum level. Furthermore, invariance of the sign of  $k^0$  allows the covariant decomposition of the tachyon field on the creation and annihilation part, so the Fock procedure can be applied. For completeness, we give also  $k^0$ ,  $\vec{k}$  and  $\underline{k}$  in terms of  $\xi$ ,  $s$ ,  $\varepsilon$

$$k^0 = \frac{\kappa}{\sqrt{(\xi-1)[(2s-1)\xi+1]}}, \quad (44a)$$

$$\vec{k} = \frac{\vec{n}\kappa\xi s}{\sqrt{(\xi-1)[(2s-1)\xi+1]}}, \quad (44b)$$

$$\underline{k} = \frac{-\vec{n}\kappa\xi s + \vec{c}\kappa\varepsilon [1 + \xi(s-1)]}{\sqrt{(\xi-1)[(2s-1)\xi+1]}}. \quad (44c)$$

Here  $\vec{c} = \frac{\vec{\sigma}}{\sigma}$ . Notice that

$$|\underline{k}| > \lim_{\xi \rightarrow \infty} |\underline{k}| = \kappa \sqrt{\frac{1 + (\varepsilon^2 - 2)\varepsilon^2 \cos^2 \theta}{1 - \varepsilon^2 \cos^2 \theta}}, \quad (45)$$

where  $\cos \theta = \vec{n}\vec{c}$ .

Finally, let us reexamine the problem of the so called ‘‘transcendental’’ tachyon. To do this, recall the transformation law for velocities in the EP synchronization [34]

$$\vec{v}'_E = \frac{\gamma_E \vec{v}_E + \vec{V}_E \left[ \frac{\vec{V}_E \vec{v}_E}{c^2} (\gamma_E + 1)^{-1} - 1 \right]}{1 - \frac{\vec{V}_E \vec{v}_E}{c^2}}, \quad (46)$$

TABLE I. Comparison of the special relativity descriptions of kinematics in the Einstein–Poincaré and Chang–Tangherlini synchronization schemes.

Synchronization scheme → Description of ↓	Einstein–Poincaré	Chang–Tangherlini
BRADYONS $k^2 = \kappa^2$ and LUXONS $k^2 = 0$	Consistent causal kinematics, fully equivalent to the CT description	Consistent causal kinematics, fully equivalent to the EP description
uniwersal notion of causality	no	yes ( $\varepsilon(dx^0) = \text{inv}$ )
covariant initial conditions	no	yes ( $x^0 = \text{const}$ )
TACHYONS $k^2 = -\kappa^2$	no	yes ( $\varepsilon(k^0) = \text{inv}$ )
covariant lower bound of energy	no ( $k_0 \rightarrow -\infty$ )	yes ( $k_{0\min} > -\infty$ )
paradox of “transcendental” tachyons	inconsistency (discontinuity)	consistent (continuous) picture

where  $\gamma_E = \sqrt{1 - \left(\frac{\vec{V}_E}{c}\right)^2}$ .

We observe that the denominator of the above transformation rule can vanish for  $|\vec{v}_E| > c$ ; Thus a tachyon moving with  $c < |\vec{v}_E| < \infty$  can be converted by a finite Lorentz map into a “transcendental” tachyon with  $|\vec{v}'_E| = \infty$ . This discontinuity is an apparent inconsistency of this transformation law; namely *tachyonic velocity space does not constitute a representation space for the Lorentz group!*

On the other hand, in the CT scheme, the corresponding transformation rule for velocities follows directly from eq. (34) and reads

$$\vec{v}' = \gamma^{-1}\vec{v} + \gamma^{-2}\vec{V} \left[ \frac{\vec{V}\vec{v}}{c^2 \left( \gamma + \sqrt{\gamma^2 + \frac{\vec{V}^2}{c^2}} \right)} - \frac{\vec{\sigma}\vec{v}}{c^2} \gamma_0^{-2} - 1 \right], \quad (47)$$

where  $\gamma = \gamma(\vec{V})$ . Thus contrary to eq. (46), the transformation law (47) is continous, does not “produce” “transcendental” tachyons and completed by rotations, forms (together with the mapping  $\vec{\sigma} \rightarrow \vec{\sigma}'$ ) a realization of the Lorentz group.

We finish this section with a Table I summarizing our results.

### C. Synchronization group and the relativity principle

From the foregoing discussion we see that the CT synchronization prefers a privileged frame corresponding to the value  $\vec{\sigma} = 0$  (*relativistic ether*). It is clear that if we forget about tachyons such a preference is only formal; namely we can choose each inertial frame as a preferred one.

Let us consider two CT synchronization schemes, say  $A$  and  $B$ , under two different choices of privileged inertial frames, say  $\Sigma_A$  and  $\Sigma_B$ . Now, in each inertial frame  $\Sigma$  two coordinate charts  $x_A$  and  $x_B$  can be introduced, according to both schemes  $A$  and  $B$  respectively. The interrelation is given by the almost obvious relations

$$x_B = T(u_E^B)T^{-1}(u_E^A)x_A, \quad (48a)$$

$$u_E^B = \Lambda_{BA}u_E^A, \quad (48b)$$

where  $u_E^A(u_E^B)$  is the four-velocity of  $\Sigma_A(\Sigma_B)$  with respect to  $\Sigma$  expressed in the EP synchronization for convenience.  $T(u_E)$  is given by the eq. (20) (with  $\vec{\sigma}_E$  replaced by the corresponding four-velocity  $u_E$ ). We observe that a set of all possible four-velocities  $u_E$  is related by Lorentz group transformations too, i.e.  $\{\Lambda_{BA}\} = L_S$ . Of course it does not coincide with our intersystemic Lorentz group  $L$ . We call the group  $L_S$  a synchronization group [22].

Now, if we compose the transformations (1, 2) of  $L$  and (48) of  $L_S$  we obtain

$$(\Lambda_S, \Lambda) : \left. \begin{aligned} x' &= T(\Lambda_S \Lambda u_E) \Lambda T^{-1}(u_E) x, \\ u'_E &= \Lambda_S \Lambda u_E \end{aligned} \right\} \quad (49)$$

with  $\Lambda_S \in L_S, \Lambda \in L$ .

Thus the composition law for  $(\Lambda_S, \Lambda)$  reads

$$(\Lambda'_S, \Lambda')(\Lambda_S, \Lambda) = (\Lambda'_S(\Lambda' \Lambda_S \Lambda'^{-1}), \Lambda' \Lambda). \quad (50)$$

Therefore, in a natural way, we can select three subgroups:

$$L = \{(I, \Lambda)\}, \quad L_S = \{(\Lambda_S, I)\}, \quad L_0 = \{(\Lambda_S, \Lambda_S^{-1})\}.$$

By means of (50) it is easy to check that  $L_0$  and  $L_S$  commute. Therefore the set  $\{(\Lambda_S, \Lambda)\}$  is simply the direct product of two Lorentz groups  $L_0 \otimes L_S$ . The intersystemic Lorentz group  $L$  is the diagonal subgroup in this direct product. From the composition law (50) it follows that  $L$  acts as an automorphism group of  $L_S$ .

Now, the synchronization group  $L_S$  realizes in fact the relativity principle. In our language the relativity principle can be formulated as follows: *Any inertial frame can be chosen as a preferred frame.* What happens, however, when the tachyons do exist? In that case the relativity principle is obviously broken: *If tachyons exist then one and only one inertial frame must be a preferred frame.* Moreover, the one-way light velocity becomes a real, measured physical quantity because conventionality thesis breaks down. It means that the synchronization group  $L_S$  is broken to the  $SO(3)_u$  subgroup (stability group of  $u_E$ ); indeed, transformations from the  $L_S/SO(3)_u$  do not leave the causality notion invariant. As we show later, on the quantum level we have to deal with *spontaneous breaking* of  $L_S$  to  $SO(3)$ .

#### IV. QUANTIZATION

As was mentioned in Section III, the following two facts, true only in the CT synchronization, are extremely important for a proper quantization procedure; namely the invariance of the sign  $\varepsilon(k^0)$  of  $k^0$  and the existence of a covariant lower energy bound. The first one allows an invariant decomposition of the tachyonic field into a creation and an annihilation parts. The second one is necessary to guarantee stability of the quantum theory. Recall that the noninvariance of  $\varepsilon(k_E^0)$  and the absence of a lower bound for  $k_{0E}$  were the main reason why the construction of a quantum theory for tachyons in the EP synchronization scheme [4,5,27] was impossible.

This section is related to the approach presented in [22].

##### A. Local tachyonic field and its plane-wave decomposition

Let us consider a hermitean, scalar field  $\varphi(x, \vec{\sigma})$  satisfying the corresponding Klein–Gordon equation with imaginary “mass”  $i\kappa$ , i.e.

$$(g^{\mu\nu}(\vec{\sigma})\partial_\mu\partial_\nu - \kappa^2)\varphi(x, \vec{\sigma}) = 0. \quad (51)$$

Our field  $\varphi$  is  $\vec{\sigma}$ -dependent because (51) is assumed to be valid for an observer in an inertial frame moving with respect to the privileged frame. Now, as in the standard case, let us consider the Lorentz-invariant measure

$$d\mu(k, \vec{\sigma}) = \theta(k^0)\delta(k^2 + \kappa^2)d^4k. \quad (52)$$

Notice, that  $d\mu$  does not have an analog in the EP synchronization because of noninvariance of the sign of  $k_E^0$  in this case.

The Heaviside step function  $\theta(k^0)$  guarantees the positivity of  $k^0$  and the lower bound of energy  $k_0$  while  $\delta(k^2 + \kappa^2)$  projects on the  $\kappa^2$ -eigenspace of the d'Alembertian  $g^{\mu\nu}\partial_\mu\partial_\nu$ . For this reason we can expand invariantly the field  $\varphi$  into the positive and negative frequencies with respect to  $k^0$

$$\varphi(x, \vec{\sigma}) = \frac{1}{(2\pi)^{3/2}} \int d\mu(k, \vec{\sigma}) (e^{ikx} a^\dagger(k, \vec{\sigma}) + e^{-ikx} a(k, \vec{\sigma})). \quad (53)$$

Integrating with respect to  $k_0$  we obtain

$$\varphi(x, \vec{\sigma}) = \frac{1}{(2\pi)^{3/2}} \int_{\Gamma} \frac{d^3 \underline{k}}{2\omega_k} (e^{ik_+x} a^\dagger(k_+, \vec{\sigma}) + e^{-ik_+x} a(k_+, \vec{\sigma})). \quad (54)$$

Here  $k_+x = k_{0+}x^0 + \underline{k}\vec{x}$  and  $k_{0+}$  is a solution of the dispersion relation  $k^2 = -\kappa^2$ ; namely  $k_{0+}$ , given in terms of velocities by the eq. (43), has the following form with respect to  $\underline{k}$

$$k_{0+} = -\frac{\vec{\sigma}}{c}\underline{k} + \gamma_0^2 \omega_k \quad (55)$$

with

$$\omega_k = \gamma_0^{-2} \sqrt{\left(\frac{\vec{\sigma}\underline{k}}{c}\right)^2 + (|\underline{k}|^2 - \kappa^2) \gamma_0^2}. \quad (56)$$

Note that  $k_+^0 = \omega_k$ . The integration range  $\Gamma$  is determined by the constraint  $k^2 = -\kappa^2$ , namely

$$|\underline{k}| \geq \kappa \left( 1 + (\gamma_0^2 - 1) \left( \frac{\vec{\sigma}\underline{k}}{|\vec{\sigma}||\underline{k}|} \right)^2 \right)^{-1/2}, \quad (57)$$

i.e. values of  $\underline{k}$  lie outside the oblate spheroid with halfaxes  $a = \kappa$  and  $b = \kappa\gamma_0^{-1}$ . Note that  $\Gamma$  is invariant under the inversion  $\underline{k} \rightarrow -\underline{k}$ .

For the operators  $a$  and  $a^\dagger$  we postulate the canonical commutation rules

$$[a(k_+, \vec{\sigma}), a(p_+, \vec{\sigma})] = [a^\dagger(k_+, \vec{\sigma}), a^\dagger(p_+, \vec{\sigma})] = 0, \quad (58a)$$

$$[a(k_+, \vec{\sigma}), a^\dagger(p_+, \vec{\sigma})] = 2\omega_k \delta(\underline{k} - \underline{p}). \quad (58b)$$

The vacuum  $|0\rangle$  is assumed to satisfy the conditions

$$\langle 0|0\rangle = 1 \quad \text{and} \quad a(k_+, \vec{\sigma})|0\rangle = 0. \quad (59)$$

By the standard procedure, using eq. (54), we obtain the commutation rule for  $\varphi(x, \vec{\sigma})$  valid for an arbitrary separation

$$[\varphi(x, \vec{\sigma}), \varphi(y, \vec{\sigma})] = -i\Delta(x - y, \vec{\sigma}), \quad (60)$$

where the analogon of the Schwinger function reads

$$\Delta(x, \vec{\sigma}) = \frac{-i}{(2\pi)^3} \int d^4k \delta(k^2 + \kappa^2) \varepsilon(k^0) e^{ikx}. \quad (61)$$

It is remarkable that  $\Delta$  does not vanish for a space-like separation which is a direct consequence of the faster-than-light propagation of the tachyonic quanta. Moreover  $\Delta(x, \vec{\sigma})|_{x^0=0} = 0$  and therefore no interference occurs between two measurements of  $\varphi$  at an instant time. This property is consistent with our interpretation of instant-time hyperplanes as the initial ones.

Now, because of the absolute meaning of the arrow of time in the CT synchronization we can introduce an invariant notion of the time-ordered product of field operators. In particular the tachyonic propagator

$$\Delta_F(x - y, \vec{\sigma}) = -i \langle 0| T(\varphi(x, \vec{\sigma}) \varphi(y, \vec{\sigma})) |0\rangle$$

is given by

$$\Delta_F(x, \vec{\sigma}) = -\theta(x^0) \Delta^-(x, \vec{\sigma}) + \theta(-x^0) \Delta^+(x, \vec{\sigma}) \quad (62)$$

with

$$\Delta^\pm(x, \vec{\sigma}) = \frac{\mp i}{(2\pi)^3} \int d^4k \theta(\pm k^0) \delta(k^2 + \kappa^2) e^{ikx}. \quad (63)$$

The above singular functions are well defined as distributions on the space of “well behaved” solutions of the Klein-Gordon equation (51).

The role of the Dirac delta plays the generalized function

$$\delta_{\Gamma}^4(x-y) = \frac{1}{(2\pi)^3} \delta(x^0-y^0) \int_{\Gamma} d^3\vec{k} e^{i\vec{k}(\vec{x}-\vec{y})}. \quad (64)$$

The above form of  $\delta_{\Gamma}^4(x)$  express impossibility of the localization of tachyonic quanta. In fact, the tachyonic field does not contain modes with momentum  $\vec{k}$  inside the spheroid defined in eq. (57). Consequently, by the Heisenberg uncertainty relation, an exact localization of tachyons is impossible.

Note also that

$$\partial^0 \Delta(x-y, \vec{\sigma}) \delta(x^0-y^0) = \delta_{\Gamma}^4(x-y)$$

so the equal-time canonical commutation relations for  $\varphi(x, \vec{\sigma})$  and its conjugate momentum  $\pi(x, \vec{\sigma}) = \partial^0 \varphi(x, \vec{\sigma})$  have the correct form

$$\delta(x^0-y^0) [\varphi(x, \vec{\sigma}), \varphi(y, \vec{\sigma})] = \delta(x^0-y^0) [\pi(x, \vec{\sigma}), \pi(y, \vec{\sigma})] = 0, \quad (65a)$$

$$[\varphi(x, \vec{\sigma}), \pi(y, \vec{\sigma})] \delta(x^0-y^0) = i\delta_{\Gamma}^4(x-y) \quad (65b)$$

as the operator eqations in the space of states.

To do the above quantization procedure mathematically more precise, we can use wave packets rather than the plane waves. Indeed, with a help of the measure (52) we can define the Hilbert space  $H_{\vec{\sigma}}^+$  of one particle states with the scalar product

$$(f, g)_{\vec{\sigma}} = \int d\mu(k, \vec{\sigma}) f^*(k, \vec{\sigma}) g(k, \vec{\sigma}) < \infty. \quad (66)$$

Now, using standard properties of the Dirac delta we deduce

$$(f, g)_{\vec{\sigma}} = \int_{\Gamma} \frac{d^3\vec{k}}{2\omega_k} f^*(k_+, \vec{\sigma}) g(k_+, \vec{\sigma}). \quad (67)$$

It is remarkable that for  $\xi \rightarrow \infty$ ,  $\omega_k \rightarrow 0$ , so to preserve inequality  $\|f\|_{\vec{\sigma}}^2 < \infty$ , the wave packets  $f(k_+, \vec{\sigma})$  rapidly decrease to zero with  $\xi \rightarrow \infty$ . This means physically that probability of “momentum localization” of a tachyon in the infinite velocity limit is going to zero in agreement with our intuition. As usually we introduce the smeared operators

$$a(f, \vec{\sigma}) = (2\pi)^{-3/2} \int d\mu(k, \vec{\sigma}) a(k, \vec{\sigma}) f^*(k, \vec{\sigma}) \quad (68)$$

and the conjugate ones. The canonical commutation rules (58) take the form

$$[a(f, \vec{\sigma}), a(g, \vec{\sigma})] = [a^{\dagger}(f, \vec{\sigma}), a^{\dagger}(g, \vec{\sigma})] = 0, \quad (69a)$$

$$[a(f, \vec{\sigma}), a^{\dagger}(g, \vec{\sigma})] = (f, g)_{\vec{\sigma}}. \quad (69b)$$

We have also  $a(f, \vec{\sigma}) |0\rangle = 0$  and  $\langle f, \vec{\sigma} | g, \vec{\sigma} \rangle = (f, g)_{\vec{\sigma}}$ , where  $|f, \vec{\sigma}\rangle = a^{\dagger}(f, \vec{\sigma}) |0\rangle$ . Let us discuss the implementation of the intersystemic Lorentz group  $L$  on the quantum level. According to our assumption of scalarity of  $\varphi(x, \vec{\sigma})$

$$L \ni \Lambda : \varphi'(x', \vec{\sigma}') = \varphi(x, \vec{\sigma}). \quad (70)$$

where  $x'$  and  $\vec{\sigma}'$  are given by (34-35). The transformation law should be realized by a representation  $U(L)$  as follows

$$U(\Lambda) \varphi(x, \vec{\sigma}) U^{-1}(\Lambda) = \varphi(x', \vec{\sigma}'), \quad (71)$$

i.e.

$$U(\Lambda) a(k, \vec{\sigma}) U^{-1}(\Lambda) = a(k', \vec{\sigma}') \quad (72)$$

and

$$U(\Lambda) |0\rangle = |0\rangle. \quad (73)$$

Therefore the wave packets must satisfy the scalarity condition (70) i.e.

$$f'(k', \vec{\sigma}') = f(k, \vec{\sigma}). \quad (74)$$

It follows that the family  $\{U(\Lambda)\}$  forms an unitary representation of  $L$ ; indeed we see that

$$(f', g')_{\vec{\sigma}'} = (f, g)_{\vec{\sigma}}. \quad (75)$$

Summarizing, the Lorentz group  $L$  is realized by a family of unitary mappings in the following bundle of Hilbert spaces

$$\begin{aligned} & H_0 \text{ (vacuum);} \\ & H_{\vec{\sigma}}^+ \text{ (bundle of one-particle spaces of states);} \\ & H_{\vec{\sigma}}^+ \otimes H_{\vec{\sigma}}^+ \text{ (bundle of two-particle spaces of states);} \\ & \dots\dots\dots \text{ etc.} \end{aligned}$$

i.e.  $H^+ = H_0 \oplus \bigcup_{\vec{\sigma}} H_{\vec{\sigma}}^+ \oplus \bigcup_{\vec{\sigma}} H_{\vec{\sigma}}^+ \otimes H_{\vec{\sigma}}^+ \oplus \dots$  etc. with the base space as the velocity space ( $\vec{\sigma}$ -space). Now we introduce wave-packet solutions of the Klein-Gordon equation *via* the Fourier transformation

$$\mathcal{F}(x, \vec{\sigma}) = (2\pi)^{-3/2} \int d\mu(k, \vec{\sigma}) f(k, \vec{\sigma}) e^{-ikx}. \quad (76)$$

In terms of these solutions the scalar product (66) reads

$$(\mathcal{F}, \mathcal{G})_{\vec{\sigma}} = -i \int d^3\vec{x} \mathcal{F}^*(x, \vec{\sigma}) \overset{\leftrightarrow 0}{\partial} \mathcal{G}(x, \vec{\sigma}). \quad (77)$$

It is easy to see that for an orthonormal basis  $\{\Phi_{\alpha}(x, \vec{\sigma})\}$  in  $H_{\vec{\sigma}}^+$  the completeness relation holds

$$\sum_{\alpha} \Phi_{\alpha}^*(x, \vec{\sigma}) \Phi_{\alpha}(y, \vec{\sigma}) = i\Delta_T^+(x - y, \vec{\sigma}), \quad (78)$$

where  $\Delta^+$  has the form (63) and it is the reproducing kernel in  $H_{\vec{\sigma}}^+$  i.e.

$$(i\Delta^+(x), \Phi)_{\vec{\sigma}} = \Phi(x, \vec{\sigma}).$$

Finally, translational invariance implies the following, almost standard, form of the fourmomentum operator

$$P_{\mu} = \int d\mu(k, \vec{\sigma}) k_{\mu} a^{\dagger}(k, \vec{\sigma}) a(k, \vec{\sigma}). \quad (79)$$

It is evident that the vacuum  $|0\rangle$  has zero fourmomentum. Furthermore,  $P_{\mu}$  applied to one-particle state  $a^{\dagger}(k_+, \vec{\sigma}) |0\rangle = |k_+, \vec{\sigma}\rangle$  gives

$$P_{\mu} |k_+, \vec{\sigma}\rangle = k_{\mu+} |k_+, \vec{\sigma}\rangle \quad (80)$$

Recall that the asymptotic one-particle lower energy bound

$$\lim_{\xi \rightarrow \infty} k_{0+} = \frac{s-1}{\sqrt{2s-1}},$$

although can be negative, it is always finite because

$$\frac{-\varepsilon}{\sqrt{1-\varepsilon^2}} < \frac{s-1}{\sqrt{2s-1}} < \frac{\varepsilon}{1-\varepsilon^2}$$

where  $\varepsilon = \frac{\sigma_E}{c} \in (0, 1)$  is fixed in a fixed inertial frame. Moreover, the average over all directions of the minimal asymptotic energy is equal exactly to zero. Thus we have constructed a consistent quantum field theory for the hermitean, scalar tachyon field  $\varphi(x, \vec{\sigma})$ . We conclude, that a proper framework to do this is the CT synchronization scheme.

## B. Spontaneous breaking of the synchronization group

As we have seen in the foregoing section, intersystemic Lorentz group is realized unitarily on the quantum level. In this section we will analyze possibility of incorporation of the synchronization group  $L_S$  in our scheme.

As was stressed in the Sec. III C, if tachyons exist then one and only one inertial frame is the preferred frame. In other words the relativity principle is broken in this case: tachyons distinguish a fixed synchronization scheme from the family of possible CT synchronizations. Consequently, because all admissible synchronizations are related by the group  $L_S$ , it should be broken. To see this let us consider transformations belonging to the  $L_0$  subgroup (see Sec. III C). They are composed from the transformations of intersystemic Lorentz group  $L$  and the synchronization group  $L_S$ ; namely they have the following form (see eq. (49) and the definition of  $L_0$ ),

$$\vec{\sigma}' = \vec{\sigma}, \quad x' = T(\vec{\sigma})\Lambda_S^{-1}T^{-1}(\vec{\sigma})x \equiv \Lambda_S^{-1}(\vec{\sigma})x. \quad (81)$$

We search an operator  $W(\Lambda)$  implementing (81) on the quantum level; namely

$$\varphi'(x, \vec{\sigma}) = W(\Lambda_S)\varphi(x, \vec{\sigma})W^\dagger(\Lambda_S) = \varphi(x', \vec{\sigma}) \quad (82)$$

This means that we should compare both sides of (82) i.e.

$$\begin{aligned} & \int d\mu(k, \vec{\sigma}) [e^{ikx} a'^\dagger(k, \vec{\sigma}) + e^{-ikx} a'^\dagger(k, \vec{\sigma})] \\ &= \int d\mu(p, \vec{\sigma}) [e^{ipx'} a^\dagger(p, \vec{\sigma}) + e^{-ipx'} a^\dagger(p, \vec{\sigma})], \end{aligned} \quad (83)$$

where  $x'$  is given by eq. (81), while, formally

$$a' = WaW^\dagger, \quad a'^\dagger = Wa^\dagger W^\dagger. \quad (84)$$

Taking into account the form of the measure  $d\mu$  (eq. (52)) and the fact that  $\Lambda_S(\vec{\sigma})$  does not leave invariant the sign of  $k^0$ , after some calculations, we deduce the following form of  $W$ :

$$a'(k, \vec{\sigma}) = \theta(k'^0)a(k', \vec{\sigma}) + \theta(-k'^0)a^\dagger(-k', \vec{\sigma}), \quad (85a)$$

$$a'^\dagger(k, \vec{\sigma}) = \theta(k'^0)a^\dagger(k', \vec{\sigma}) + \theta(-k'^0)a(-k', \vec{\sigma}), \quad (85b)$$

where  $k' = \Lambda_S(\vec{\sigma})k$ .

We see that formally unitary operator  $W(\Lambda_S)$  is realized by the Bogolubov-like transformations; the Heaviside  $\theta$ -step functions are the Bogolubov coefficients. The form (85) of the transformations of the group  $L_0$  reflects the fact, that a possible change of the sign of  $k^0$  causes a different decomposition of the field  $\phi$  on the positive and negative frequencies. Furthermore it is easy to check that the transformation (85) preserves the canonical commutation relations (58).

However, the formal operator  $W(\Lambda_S)$  realized in the ring of the field operators, cannot be unitarily implemented in the space of states in general; only if  $\Lambda = \Lambda_u$  is an element of the stability group  $SO(3)_u$  of  $u$  in  $L_S$ , it can be realized unitarily. This is related to the fact that  $\Lambda_u(\vec{\sigma})$  does not change the sign of  $k^0$  for any  $k$ . Indeed, notice firstly that for  $\Lambda_S \in L_S/SO(3)$ ,  $W(\Lambda_S)$  does not annihilate the vacuum  $|0\rangle$ . Moreover, the particle number operator

$$N = \int d\mu(k, \vec{\sigma}) a^\dagger(k, \vec{\sigma}) a(k, \vec{\sigma}) \quad (86)$$

applied to the “new” vacuum

$$|0\rangle' = W^{-1}|0\rangle \quad (87)$$

gives

$$N|0\rangle' = \delta^3(0) \int_{\Gamma} d^3\underline{k} \theta(-(\Lambda_S(\vec{\sigma})k_+)^0) |0\rangle'. \quad (88)$$

The right side of the above expression diverges like  $\delta^6(0)$  for any  $\Lambda_S(\vec{\sigma}) \in L_S/SO(3)_u$ . Only for the stability subgroup  $SO(3)_u \subset L_S$  vacuum remains invariant. Thus, a “new” vacuum  $|0\rangle'$ , related to an essentially new synchronization,

contains an infinite number of “old” particles. As is well known, in such a case, two Fock spaces  $H$  and  $H'$ , generated by creation operators from  $|0\rangle$  and  $|0'\rangle$  respectively, cannot be related by a unitary transformation<sup>4</sup> ( $W(\Lambda_S)$  in our case). Therefore, we have deal with the so called spontaneous symmetry breaking of  $L_S$  to the stability subgroup  $SO(3)$ . This means that physically privileged is only one realization of the canonical commutation relations (58) corresponding to a vacuum  $|0\rangle$  defined by eq. (59). Such a realization is related to a definite choice of the privileged inertial frame and consequently to a definite CT synchronization scheme. Thus we can conclude that *tachyons distinguish a preferred frame via spontaneous breaking of the synchrony group*.

To complete discussion, let us apply the fourmomentum operator  $P_\mu$  to the new vacuum  $|0'\rangle$ . As the result we obtain

$$P_\mu |0'\rangle = -\delta^3(0) \Lambda_{S\mu}{}^\nu(\vec{\sigma}) \int_\Gamma d^3\vec{k} \theta(-(\Lambda_S(\vec{\sigma})k_+)^0) k_\nu |0'\rangle. \quad (89)$$

This expression diverges again like  $\delta^7(0)$  for  $\Lambda_S \in L_S/SO(3)_u$ . Therefore a transition to a new vacuum ( $\equiv$  change of the privileged frame) demands an infinite momentum transfer, i.e. it is physically unadmissible. This last phenomenon supports our claim that existence of tachyons is associated with spontaneous breaking of the the synchronization group.

On the other hand it can be simply shown [22] that a free field theory for standard particles (bradyons or luxons), formulated in CT synchronization, is unitarily equivalent to the standard field theory in the EP synchronization.

## V. CONCLUSIONS

We can conclude that, contrary to the current opinion, a consistent quantization of the tachyonic field in the framework of special relativity is possible and it is closely connected with the choice of an appropriate synchronization scheme. From this point of view the Einstein–Poincaré synchronization is useless in the tachyonic case. On the other hand, in a description of bradyons and luxons only, we are free in the choice of a synchronization scheme. For this reason we can use in this case CT-synchronization as well as the standard one.

The CT-synchronization, a natural one for a description of tachyons, favours a reference frame (privileged frame). This preference is only formal if tachyons do not exist. However, if they exist, then an inertial reference frame is really (physically) preferred. As a consequence, the one-way light velocity can be measured in this case and, in general, it will be direction-dependent for a moving observer. Light velocity is isotropic only in the privileged frame.

A next step is to construct a field theory for a fermionic tachyon and a local interaction with other fields.

Two questions arise immediately. Do tachyons exist? Are candidates from the family of observable particles to be tachyons?

An answer to the above questions is not so simple. For example, about the existence of tachyons we can deduce indirectly. Indeed, we know that they prefer an inertial frame (relativistic ether). On the other hand we have in the real world a serious candidate to such a frame; namely frame related to the background radiation. Moreover, the standard cosmological model and related models possess in fact an absolute time (radius of the universe). Furthermore, let us notice that if the one-way velocity is really anisotropic, then our present interpretation of the experimental data about of distribution of cosmic matter, its anisotropy and anisotropy of the background radiation, is in some sense false and demands a reinterpretation. Indeed, our actual picture assumes isotropic one-way light velocity equal to  $c$ . Maybe, by means of anisotropy of the one-way light velocity it is possible to obtain a more isotropic picture of the world.

Of course, indirect arguments are not decisive ones. An experimental evidence for tachyons can be a decisive argument only. However, up to now, in the current opinion, no candidates to be a tachyon. Notwithstanding, there is a number of experiments [6,7] suggesting that the muonic neutrino mass square is negative by a few standard deviations. The experimental data of the electron neutrino mass square, although not so dramatic, also prefers a negative value [6]. Maybe some decisive data will be obtained after more accurate measurements of the charged pion mass which have began now at the Paul Scherrer Institute [6,7]. If the future results will support intriguing hypothesis by Chodos *et al.* that neutrino is a fermionic tachyon, then a modification of the theory of electroweak interactions will be necessary.

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<sup>4</sup>We can treat (85), in some sense, as a quantum version of the familiar *reinterpretation principle* [21]. We find that the reinterpretation principle cannot be unitarily implemented.



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